Energy loss of a heavy quark produced in a finite size medium

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ABSTRACT: We study the medium-induced energy loss $-\Delta E_0(L_p)$ suffered by a heavy quark produced at initial time in a quark-gluon plasma, and escaping the plasma after travelling the distance L_p. The heavy quark is treated classically, and within the same framework $-\Delta E_0(L_p)$ consistently includes: the loss from standard collisional processes, initial bremsstrahlung due to the sudden acceleration of the quark, and transition radiation. The radiative loss *induced by rescatterings* $-\Delta E_{\rm rad}(L_p)$ is not included in our study. For a ultrarelativistic heavy quark with momentum $p \gtrsim 10\,{\rm GeV}$, and for a finite plasma with $L_p \lesssim 5$ fm, the loss $-\Delta E_0(L_p)$ is strongly suppressed compared to the stationary collisional contribution $-\Delta E_{\rm coll}(L_p) \propto L_p$. Our results support that $-\Delta E_{\rm rad}$ is the dominant contribution to the heavy quark energy loss (at least for $L_p \lesssim 5$ fm), as indeed assumed in most of jet-quenching analyses. However they might raise some question concerning the RHIC data on large $p_\perp$ electron spectra.

KEYWORDS: QCD, Jets.

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Contents

1. Introduction

The attenuation of jets — jet quenching — was initially suggested by Bjorken [1] as a possible signature of the quark-gluon plasma (QGP). Since, the suppression of hadron spectra at large transverse momentum $p_\perp$, in ultrarelativistic heavy ion compared to proton-proton collisions, has been observed [2, 3] at the Relativistic Heavy Ion Collider (RHIC). Until recently this suppression (and other features of jet quenching) seemed consistent [4] with the energy loss $-\Delta E_{\rm rad}$ of the parent parton due to gluon radiation *induced by its rescatterings* [5, 6] in the hot or dense medium created in the heavy ion collision.

However, recent experimental data on the $p_\perp$-spectra of electrons arising from heavy flavour decays [7, 8], when compared to theory [9], suggest that heavy quarks lose more energy than $-\Delta E_{\rm rad}$ only.[1] It was suggested in ref. [11] that this 'single electron puzzle' might be solved by the *collisional* energy loss, previously neglected in the total parton energy loss. However, one should remember that other sources of medium-induced energy loss are usually neglected, namely the Ter-Mikayelian (TM) effect and transition radiation. Thus it would be incorrect to invoke collisional loss without studying the effect of the latter radiative contributions.[2] Transition radiation and the TM effect have already been studied, for instance in refs. [12, 13] and [14] respectively. In this paper we present a theoretical model allowing to treat consistently the *zeroth order* of the (heavy quark) energy loss in an opacity expansion [6], denoted as $-\Delta E_0$, which includes the TM effect, transition radiation,

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[1]It is stressed in ref. [10] that this statement might be somewhat premature, since the theoretical calculation of heavy quark production suffers from large uncertainties already in proton-proton collisions, and also because the contributions to the electron spectra from charm and beauty are not yet separated experimentally.

[2]We stress again that the TM effect and transition radiation are radiative contributions distinct from that induced by rescatterings $-\Delta E_{\rm rad}$.

and collisional energy loss. Our main result is shown in figure 3. For $L_p \lesssim 5$ fm we find that $-\Delta E_0$ is suppressed compared to the collisional loss $-\Delta E_{\mathrm{coll}}$, indicating that $-\Delta E_{\mathrm{rad}}$ should be dominant for such values of the plasma size (contrary to what is suggested in [11]), and leaving the 'single electron puzzle' unsolved.

In section 2 we present our theoretical model, which is inspired from that of ref. [15], in the case of an infinite medium. We assume the heavy quark to be produced in a hard subprocess *at initial time*, in a static QGP of high temperature T and small coupling $g \ll 1$. The latter hypothesis implies the hierarchy $1/T \ll r_D \ll \lambda$, where $1/T$ is the average distance between two constituents of the QGP, $r_D = 1/m_D \sim 1/(gT)$ is the Debye radius and $\lambda \sim 1/(g^2 T)$ is the mean free path of the heavy quark. Under these hypotheses, we can describe the QGP via its *collective* response to the current [16], where the (longitudinal and transverse) dielectric functions are obtained from the gluon polarization tensor. In [15] we studied the *mechanical work* W undergone by the heavy quark when travelling in the (infinite) plasma, and used the hard thermal loop (HTL) [17] approximation for the dielectric functions. In section 2 we show that to a very good accuracy, the model of [15] can be simplified by assuming an isotropic plasma described by the dielectric function (2.8). In fact, in the ultrarelativistic limit $\gamma = E/M = 1/\sqrt{1-v^2} \to \infty$ we are considering (M, E and v being the heavy quark mass, energy and velocity), the model (2.8) reproduces *exactly* the results of [15] for the work W. At finite but large $\gamma \gg 1$, the differences between the HTL approximation and the simple model (2.8) are numerically negligible, as shown in figure 1. We end section 2 by a critical discussion. We indeed stress that in the academic case of an infinite QGP, the *mechanical work* W calculated in [15] *should not* be interpreted as an observable energy loss (contrary to what was done in [15]). This is due to the fact that the quark asymptotic ($t = +\infty$) states are different in vacuum and in an infinite QGP [18].

In section 3 we consider the more realistic case where the quark is produced in a hard subprocess in a *finite size* (but still static) QGP, and travels the distance L_p before escaping the medium. In this case the "asymptotic" quark states are the same in vacuum and in the presence of the medium, and it is legitimate to interpret the work $-W$ as the (medium-induced) quark energy loss, provided one includes transition radiation. We thus study the heavy quark energy loss $-W \equiv -\Delta E(L, L_p)$, where L is the total distance travelled by the (deconfined) quark, related to the quark hadronization time as $L = v t_{\mathrm{had}} \sim v\gamma/\Lambda_{\mathrm{QCD}}$. We assume the heavy quark to be relativistic enough so that it hadronizes outside the medium, $t_{\mathrm{had}} \simeq L \gg L_p$. We also consider a high temperature QGP, with a Debye mass $m_D \sim gT \gg \Lambda_{\mathrm{QCD}}$. The hadronization time being larger[3] than the other relevant length scales of the problem L_p and $1/m_D$, the loss $-\Delta E(L = \infty, L_p) \equiv -\Delta E_0(L_p)$ will be the main quantity of interest, affecting the final hadron (or decay electron) $p_\perp$-spectrum. The calculation of transition radiation fields in section 3 is done using the simple model (2.8) motivated in section 2, and our final result for the *zeroth order* energy loss $-\Delta E_0(L_p)$ is shown in figure 3.

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[3] For a charm quark the assumption $t_{\mathrm{had}} \gg L_p$ should be reasonable in practical applications where $L_p \lesssim 5$ fm and $E \geq 10\,\mathrm{GeV}$.

In the present study we focus on the case of a heavy quark of mass M. Comparing $-\Delta E_0(L_p)$ to the stationary collisional energy loss $-\Delta E_{\rm coll}(L_p) \propto L_p$ [19], our estimate (see (3.9) and figure 3) $(-\Delta E_{\rm coll}(L_p) + \Delta E_0(L_p))/E = \sqrt{2}/3\, C_F \alpha_s m_D/M \simeq 5\%$, for a charm quark of mass $M = 1.5\,{\rm GeV}$ and for $L_p > 4$ fm, is numerically consistent with [13, 20]. However we find explicitly that the reduction of $-\Delta E_0(L_p)$ compared to $-\Delta E_{\rm coll}(L_p)$ scales as γ when $\gamma \to \infty$, which can be interpreted as an effective 'retardation' of the stationary regime (see figure 3) scaling similarly. We think it is important to extract such an asymptotic parametric behaviour in order to get a better insight on heavy quark energy loss, and also in view of applications to heavy ion collisions at higher energies such as those planned at the Large Hadron Collider. We recall that our calculation includes, in a *unique* classical framework, transition radiation, initial bremsstrahlung and collisional processes. The latter were not included in [12] and were calculated in [20] without the contribution from transition radiation (calculated separately in [13]). Let us mention ref. [21], where an effect of interference between elastic and radiative amplitudes is studied. This effect is not included in our approach.

We stress that our results should be taken at the qualitative level only. Indeed, we work in an ideal framework, where the medium created in the heavy ion collision is assumed to be a static, equilibrated, high temperature QGP with small coupling $g \ll 1$. We also work in the fixed coupling approximation. In the case of a running α_s we expect the slope $(-dE/dx)_{\rm coll}$ of the stationary collisional loss to be independent of E when $E \to \infty$ [22], contrary to the result for fixed α_s [16, 19] which is logarithmic in E (see (3.10)). Despite this reduced energy dependence, $(-dE/dx)_{\rm coll}$ with running α_s might be larger than in the fixed coupling approximation, because it behaves as $(-dE/dx)_{\rm coll} \propto \alpha_s$ instead of $\propto \alpha_s^2$ [22, 23]. We thus cannot exclude that a larger $(-dE/dx)_{\rm coll}$ than usually assumed partly compensates the drastic suppression we find for $-\Delta E_0(L_p \lesssim 5$ fm$)$. A consistent calculation of $-\Delta E_0(L_p)$ (and of the slope of its asymptote $(-dE/dx)_{\rm coll}$) *with running α_s* is needed to answer this question.

2. Infinite medium: theoretical model and critical discussion

In this section we first recall the model used in [15] (which was directly adapted from ref. [16]) to study the mechanical work W undergone by a heavy quark produced at $t = 0$ in an infinite QGP, and travelling the distance L in the plasma. We then simplify this model in order to focus on its essential feature.

We start from the expressions of the longitudinal and transverse (chromo-)electric fields in momentum space $K = (\omega, \vec{k})$,

$$\vec{E}_L^a = \frac{4\pi}{i\omega} \frac{\vec{j}_L^a}{\epsilon_L}; \qquad \vec{E}_T^a = \frac{4\pi}{i\omega} \frac{\vec{j}_T^a}{\epsilon_T - k^2/\omega^2}, \tag{2.1}$$

which follow from Maxwell equations in a medium with dielectric functions ϵ_L and ϵ_T, in the abelian approximation and within linear response theory. For a heavy quark of color charge q^a (with $q^a q^a = C_F \alpha_s$) produced at $t = 0$, the classical 3-*vector* current density $\vec{j}^a$

is of the form [15]:

$$\vec{j}^a(K) = iq^a \frac{\vec{v}}{K.V + i\eta},$$ (2.2)

with longitudinal and transverse components given by $\vec{j}_L = (\vec{j}.\vec{k}/k^2)\vec{k}$ (where $k = |\vec{k}|$) and $\vec{j}_T = \vec{j} - \vec{j}_L$. The quark 4-velocity is denoted by $V = (1, \vec{v})$ and assumed to be constant in the following. Let us mention that we can easily form a *conserved* 4-current whose vector component is precisely given by (2.2), for instance by assuming the fast heavy quark to be produced in conjunction with a static antiquark at $t = 0$. This was done in ref. [15], where it was shown that the dominant effect (scaling as γ when $\gamma \to \infty$, see (2.11) below) on W arises from the fast quark contribution to the conserved current, given by (2.2). The precise form of the conserved current used in [15] only affects the *longitudinal* contribution to the medium-induced energy loss, which receives a term corresponding to the difference of the dipole binding energies in medium and in vacuum. When $\gamma \gg 1$ this contribution is subleading [15] compared to the dominant (*transverse*) contribution $\propto \gamma$. In the present study, where we focus on the leading effects scaling as γ, we can thus consider (2.2) as a relevant model for the current.

In coordinate space the medium-induced electric field reads

$$\vec{\mathcal{E}}^a(t, \vec{x}) = \int \frac{d^4K}{(2\pi)^4} e^{-i(\omega t - \vec{k}.\vec{x})} \left[\vec{E}_L^a + \vec{E}_T^a \right]_{\text{ind}},$$ (2.3)

where the 'induced' prescription corresponds to subtracting the vacuum $\epsilon_L = \epsilon_T = 1$ contribution. The induced mechanical work W of the electric force on the quark is given by

$$
\begin{aligned}
-W &= -\vec{v} \cdot \int_0^{L/v} dt\, q^a \vec{\mathcal{E}}^a(t, \vec{v}t) \\
&= -q^a \vec{v} \cdot \int \frac{d^4K}{(2\pi)^4} \int_0^{L/v} dt\, e^{-iK.V\,t} \left[\vec{E}_L^a + \vec{E}_T^a \right]_{\text{ind}}.
\end{aligned}
$$ (2.4)

In ref. [15] the work W is calculated by choosing for the dielectric functions ϵ_L and ϵ_T those of a high temperature QGP. The latter have a rich analytical structure, which somewhat complicates the calculation of the energy loss. In particular, the dielectric functions have a cut in the spacelike $|\omega| < k$ region (physically corresponding to Landau damping), responsible for the leading large L behaviour,

$$-W(L) \underset{L \to \infty}{\simeq} -\Delta E_{\text{coll}}(L) + \mathcal{O}\left(L^0\right) \quad \text{where} \quad -\Delta E_{\text{coll}}(L) \equiv \left(-\frac{dE}{dx}\right)_{\text{coll}} L.$$ (2.5)

We are interested in the difference, denoted as $d(L)$, between $-W(L)$ and the stationary law for collisional loss $-\Delta E_{\text{coll}}(L)$,

$$d(L) = -W(L) + \Delta E_{\text{coll}}(L) = -W(L) + \left(\frac{dE}{dx}\right)_{\text{coll}} L.$$ (2.6)

By definition the function $d(L)$ tends to a constant d_∞ when $L \to \infty$.

It has been shown [15] that for $\gamma \gg 1$, $d(L)$ arises dominantly from the transverse contribution and from the phase space region $k \sim \gamma m_D$ and $x = \omega/k \to 1$. In this limit the HTL [17] transverse gluon propagator takes a simple form,

$$\Delta_T(\omega = kx, k) = \frac{-1}{\omega^2 - k^2 - \Pi_T(x)} \equiv \frac{-1}{\epsilon_T\, \omega^2 - k^2} \underset{x \to 1}{\simeq} \frac{-1}{\omega^2 - k^2 - m^2}\,, \qquad (2.7)$$

where $m = m_D/\sqrt{2}$ is the asymptotic (transverse) gluon thermal mass and the retarded prescription $\omega \to \omega + i\eta$ is implicit. Thus $d(L)$ can be simply modelled by assuming a medium with dielectric function

$$\epsilon(\omega) = 1 - \frac{m^2}{\omega^2}\,. \qquad (2.8)$$

All medium effects are encoded in the single parameter m. As we will shortly see, the model (2.8) captures the exact leading order in $\gamma \gg 1$, and will greatly simplify the calculation of transition radiation in the next section, due to $\epsilon(\omega)$ being independent of k — corresponding to the plasma response involving no spatial dispersion.[4] The main effects included in our calculation — initial bremsstrahlung and transition radiation — will arise from the domain $\omega \simeq k \propto \gamma m$, justifying a posteriori using (2.8). We evaluate (2.4) by performing the ω-integral using Cauchy's theorem, and the function $d(L)$ by subtracting the leading term (linear in L) when $L \to \infty$,

$$d(L) = \frac{C_F \alpha_s}{\pi^2} \int d^3\vec{k} \left\{ \vec{v}_T^2 \frac{\sin^2\left[(E_k - \vec{k}\cdot\vec{v})L/(2v)\right]}{(E_k - \vec{k}\cdot\vec{v})^2} + \right.$$
$$\left. + \vec{v}_L^2 \left[\frac{\sin^2\left[(m - \vec{k}\cdot\vec{v})L/(2v)\right]}{(m - \vec{k}\cdot\vec{v})^2} - \frac{\pi L}{2v}\delta(m - \vec{k}\cdot\vec{v}) \right] \right\}_{\text{ind}}\,, \quad (2.9)$$

where $E_k = \sqrt{k^2 + m^2}$ and the 'induced' prescription corresponds to subtracting the value of the integrand at $m = 0$. In (2.9) the second line corresponds to the longitudinal contribution, which will be subleading in the following.

The explicit calculation of (2.9) yields:

$$d_\infty \equiv \lim_{L \to \infty} d(L) = -2C_F \alpha_s m \left[\frac{1}{\sqrt{1 - v^2}} - \frac{\arcsin v}{v} + \frac{\pi}{2v} \right]\,, \qquad (2.10)$$

where the last term in the brackets stands for the longitudinal contribution. In the $v \to 1$ limit, (2.10) reproduces exactly the result of [15] obtained within the HTL approximation, and arises dominantly from the transverse contribution:[5]

$$d_\infty \underset{v \to 1}{\simeq} -\sqrt{2}C_F \alpha_s m_D\, \gamma\,. \qquad (2.11)$$

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[4]The isotropy implied by (2.8) obviously imposes $\epsilon_L = \epsilon_T = \epsilon(\omega)$.

[5]We note that the longitudinal contribution given in the second line of (2.9) is not the same as in ref. [15]. In particular it does not include the induced binding energy of the dipole created at $t = 0$. This is because instantaneous ($\omega = 0$) self-interactions are suppressed in the model (2.8). These details turn out to be irrelevant since the longitudinal contribution is subleading when $\gamma \gg 1$, both in ref. [15] and in the present study.
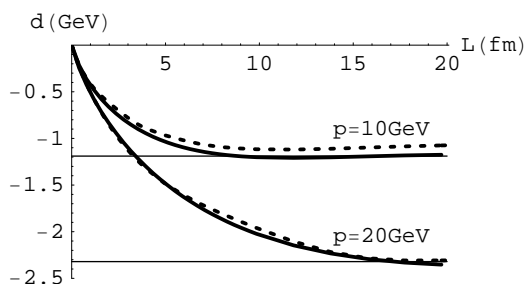
Figure 1: The function $d(L)$ defined by (2.6) and given by (2.9) in the model (2.8) (thick lines). We consider a charm quark of mass $M = 1.5\,\text{GeV}$, use $\alpha_s = 0.2$, $m^2 = m_D^2/2 = 2\pi\alpha_s T^2(1+n_f/6) \simeq (0.32\,\text{GeV})^2$ for $n_f = 2$ flavours and a plasma temperature $T = 0.25\,\text{GeV}$. We show $d(L)$ for two values of the charm quark momentum, $p = 10\,\text{GeV}$ and $p = 20\,\text{GeV}$. The thin straight lines give the values of d_∞ (see (2.10)) for the corresponding values of p. The (HTL) results of ref. [15] for $d(L)$ are given by the dashed lines.

The numerical evaluation of $d(L)$ is displayed in figure 1, where it is also compared to the results of [15] obtained with the HTL expressions of the dielectric functions. Since the results of ref. [15] for $d(L)$ are very accurately reproduced with the simple model (2.8), we will use the latter model in section 3 in order to derive the 'zeroth order' of the heavy quark energy loss $-\Delta E_0(L_p)$ in a finite QGP.

Before doing that let us discuss (see also [18]) the incorrect interpretation which was done in [15]. In [15] the stationary collisional loss $-\Delta E_{\text{coll}}(L) \propto L$ [19] was added to $d(L)$, and the result *interpreted* as the quark *energy loss* $-\Delta E(L)$ in a infinite plasma. Due to the large negative value of d_∞, a significant delay before the onset of the stationary (linear) behaviour was observed, and designated as 'retardation effect'. As noted in [15] this 'retardation effect' is not a genuine retardation of purely collisional loss, since various physical effects contribute to the apparent delay. First, part of the work done on the charge is due to the initial radiation induced by the sudden acceleration of the quark at $t = 0$. Due to the difference between the gluon dispersion relations in medium and in vacuum, initial bremsstrahlung in QGP differs from that in vacuum. This is the QCD equivalent of the Ter-Mikayelian (TM) effect, studied in [14] and also properly identified in [15]. In the limit $\gamma \gg 1$, half of the apparent 'retardation' is actually due to the TM effect [15].

As can already be seen in the case of a charge with $v \simeq 1$ produced in vacuum, initial radiation represents only *half* of the mechanical work done on the charge after its production. Using Poynting's theorem we can show that the other half is associated to the creation of the charge's proper field. Contrary to what was implicitly done in [15], this *self-energy* contribution should not be counted as 'energy loss' as it is part of the charge asymptotic state. With an accurate definition of energy loss in terms of asymptotic states [18], we find that the retardation time of *purely collisional* energy loss is quite small, $t_{\text{ret}} \sim 1/m_D$, instead of $t_{\text{ret}} \propto \gamma/m_D$ as argued in [15].

In the next section we will consider the more realistic situation, where the quark is produced in a finite size QGP. In this case the 'asymptotic' (we assume the quark to

hadronize long after escaping the medium) quark self-energy is the same as in vacuum. Interpreting (minus) the mechanical work on the quark as energy loss is then legitimate, provided transition radiation is taken into account. Let us stress here that our final result for $-\Delta E_0(L_p)$ in figure 3 displays an *effective* time delay $t_{\rm ret} \propto \gamma/m_D$ before the linear regime. This effective delay is not due to a retardation of collisional energy loss (at least at leading order in $\gamma \gg 1$), but arises from a non-compensation — already noted in [13] — between the TM effect and transition radiation.

3. Finite size medium: implementing transition radiation

Here we study how the finite size L_p of the medium affects the work W done on the heavy quark. Instead of $W(L) \equiv W(L, L_p = \infty)$, we now consider the explicit dependence of $W(L, L_p)$ on L_p, and implement transition radiation induced by the discontinuity at $L = L_p$ between medium and vacuum.

We derive transition radiation along the lines of ref. [24]. We assume that the separation surface between medium and vacuum is a plane located at $z = L_p$, and that the heavy quark produced at $t = 0$ and $z = 0$ in the medium (with the associated current (2.2)) travels along the z-axis (with $v^z = v > 0$). We denote the medium and vacuum as media 1 and 2, with dielectric functions $\epsilon_1(\omega)$ given by (2.8) and $\epsilon_2(\omega) = 1$. (This is similar to the model used in [13]). Transition radiation is obtained by adding to the fields (2.1) in both media some (transverse) fields $\vec{E}_T'$ solutions of the homogeneous (abelian-like) Maxwell equations. It is convenient [24] to express these fields in the mixed $(\omega, \vec{k}_\perp, z)$ representation,

$$\vec{E}_{T1}'^a(\omega, \vec{k}_\perp, z) = 4\pi q^a\, h_1(\omega, k_\perp)\, e^{-i\sigma_1(z-L_p)} \left(k_\perp^2 \vec{e}_z + \sigma_1 \vec{k}_\perp \right)$$
$$\vec{E}_{T2}'^a(\omega, \vec{k}_\perp, z) = 4\pi q^a\, h_2(\omega, k_\perp)\, e^{i\sigma_2(z-L_p)} \left(k_\perp^2 \vec{e}_z - \sigma_2 \vec{k}_\perp \right) \qquad (3.1)$$
$$\sigma_i = \sigma_i(\omega, k_\perp) \equiv \sqrt{\epsilon_i\, \omega^2 - k_\perp^2}\,,$$

where the subscript "$\perp$" denotes a vector component orthogonal to the unit vector $\vec{e}_z$ specifying the z-axis, the retarded prescription $\omega \to \omega + i\eta$ is implicit, and the square root is defined as having a cut along the *positive* real axis, i.e. such that $\mathrm{Im}\sqrt{z} \geq 0$ for all complex z. One directly sees that the fields (3.1) satisfy[6] the (homogeneous) Gauss and d'Alembert's equations, which read $\partial_z E_z + i\vec{k}_\perp \cdot \vec{E}_\perp = 0$ and $(\epsilon\,\omega^2 - k_\perp^2)\vec{E} + \partial_z^2\vec{E} = \vec{0}$ in the mixed representation.

With our definition of the square root we have

$$\sigma_i = \begin{cases} \mathrm{sign}(\omega)\sqrt{\epsilon_i\,\omega^2 - k_\perp^2} & \text{for } \epsilon_i\,\omega^2 > k_\perp^2 \quad \text{region (i)} \\ i\sqrt{k_\perp^2 - \epsilon_i\,\omega^2} & \text{for } \epsilon_i\,\omega^2 < k_\perp^2 \quad \text{region (ii)} \end{cases}. \qquad (3.2)$$

The electric field in coordinate space arises from either region (i) or (ii) in momentum space. The contributions from region (i) correspond to a superposition of plane waves

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[6]Recall that the permittivity does not depend on k_z in the model (2.8).

propagating with decreasing z (for $\vec{E}'_{T1}$) or increasing z (for $\vec{E}'_{T2}$), in a direction specified by the wave vector $\vec{k} = (\vec{k}_\perp, -\sigma_1)$ or $\vec{k} = (\vec{k}_\perp, \sigma_2)$. The absence of components $\propto e^{+i\sigma_1 z}$ in $\vec{E}'_{T1}$ and $\propto e^{-i\sigma_2 z}$ in $\vec{E}'_{T2}$ is dictated by the transition fields being created at the interface between medium ($z < L_p$) and vacuum ($z > L_p$), so that no wave comes from $z = -\infty$ in the medium or from $z = +\infty$ in the vacuum. The contributions from region (ii) correspond to surface waves which decay exponentially in the longitudinal distance. Including terms $\propto e^{+i\sigma_1 z}$ in $\vec{E}'_{T1}$ and $\propto e^{-i\sigma_2 z}$ in $\vec{E}'_{T2}$ would be unphysical as they diverge for $z \to -\infty$ and $z \to +\infty$ respectively. We stress that both types of waves (plane waves and surface waves) are simultaneously included in our framework by defining the square root with a cut along the positive real axis (with the implicit retarded prescription $\omega \to \omega + i\eta$).

The functions h_1 and h_2 in (3.1) are obtained by requiring that the total (inhomogeneous) fields

$$\vec{E}_1^a = \vec{E}_{L1}^a + \vec{E}_{T1}^a + \vec{E}'^a_{T1}$$
$$\vec{E}_2^a = \vec{E}_{L2}^a + \vec{E}_{T2}^a + \vec{E}'^a_{T2} \tag{3.3}$$

are consistent with the continuity of the components $\vec{E}_\perp(t, \vec{x}_\perp, z)$ and $\vec{D}^z(t, \vec{x}_\perp, z)$ at the transition surface $z = L_p$. We evaluate $h_1(\omega, k_\perp)$ and $h_2(\omega, k_\perp)$ explicitly in appendix A. The results are given by (A.8) and (A.10).

The work $-W(L, L_p)$ is given by an expression similar to (2.4), but we now have to specify whether the quark is in medium 1 or medium 2 (the vacuum) at time t:

$$-W(L, L_p) = -q^a \vec{v} \cdot \int_0^{L/v} dt \left\{ \Theta(L_p - vt) \vec{\mathcal{E}}_1^a(t, \vec{v}t) + \Theta(vt - L_p) \vec{\mathcal{E}}_2^a(t, \vec{v}t) \right\}. \tag{3.4}$$

The medium-induced fields are related to the total fields (3.3) which include the transition fields. Instead of (2.3) we have

$$\vec{\mathcal{E}}_i^a(t, \vec{x}) = \int \frac{d^4 K}{(2\pi)^4} e^{-i(\omega t - \vec{k} \cdot \vec{x})} \left[\vec{E}_{Li}^a + \vec{E}_{Ti}^a + \vec{E}'^a_{Ti} \right]_{\text{ind}}. \tag{3.5}$$

Inserting (3.5) in (3.4) and subtracting the leading term (linear in L) when $L \to \infty$ we get:
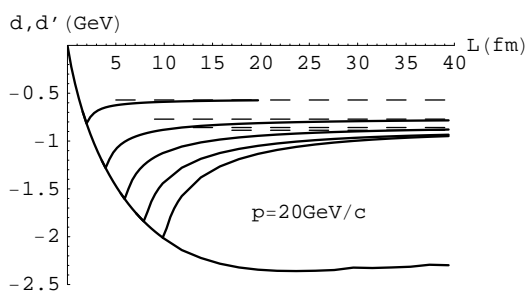
$$d(\hat{L}) - W'(L, L_p) \equiv d'(L, L_p); \qquad \hat{L} \equiv \text{Min}\{L, L_p\}, \tag{3.6}$$

where $d(L)$ is given by (2.9) and the contribution $-W'(L, L_p)$ arises from the transition fields:

$$-W'(L, L_p) = -q^a \vec{v} \cdot \int \frac{d^4 K}{(2\pi)^4} \left\{ \int_0^{\hat{L}/v} dt\, e^{-iK.Vt} \vec{E}'^a_{T1} + \right.$$
$$\left. + \Theta(L - L_p) \int_{L_p/v}^{L/v} dt\, e^{-iK.Vt} \vec{E}'^a_{T2} \right\}. \tag{3.7}$$

Using the mixed representation (3.1) of the transition fields and integrating over time we obtain:

$$-W'(L, L_p) = \frac{C_F \alpha_s}{2\pi} \int_0^\infty dk_\perp^2\, k_\perp^2 \int_{-\infty}^\infty d\omega \times$$

Figure 2: The function $d'(L, L_p)$ of (3.6) for $L_p = 2$, 4, 6, 8 and 10 fm (from top to bottom) together with the function $d(L)$ (lower curve) of figure 1, for $p = 20\,\text{GeV}$. The dashed straight lines represent the asymptote d'_∞ of $d'(L, L_p)$ for the different values of L_p.

$$\times \left\{ \frac{1 - e^{-i(\frac{\omega}{v} + \sigma_1)\hat{L}}}{\frac{\omega}{v} + \sigma_1} \, ih_1(\omega, k_\perp) \, e^{i\sigma_1 L_p} + \Theta(L - L_p) \times \right.$$

$$\left. \times \frac{e^{-i(\frac{\omega}{v} - \sigma_2)L_p} - e^{-i(\frac{\omega}{v} - \sigma_2)L}}{\frac{\omega}{v} - \sigma_2} \, ih_2(\omega, k_\perp) \, e^{-i\sigma_2 L_p} \right\}. \qquad (3.8)$$

The function $d'(L, L_p)$ defined in (3.6) is shown in figure 2. For finite L_p, $d'(L, L_p)$ has a cusp at $L = L_p$, where the effect of the transition fields on the work done on the heavy quark starts.[7] Most importantly, the asymptote d'_∞ of $d'(L, L_p)$ in the limit $L \gg L_p \to \infty$ *scales in γ* when $\gamma \to \infty$ (see (B.8)),

$$d'_\infty \underset{v \to 1}{\simeq} \frac{1}{3} d_\infty \underset{v \to 1}{\simeq} -\frac{1}{3}\sqrt{2} C_F \alpha_s m_D \gamma. \qquad (3.9)$$

As discussed in the end of section 2, in the case of a finite plasma size it is legitimate to interpret the *mechanical work* $d'(L = \infty, L_p)$ as quark *energy loss*. It is also clear that $d'(\infty, L_p)$ includes the (radiative) contributions from initial bremsstrahlung and transition radiation. In order to obtain the 'zeroth order' loss $-\Delta E(L = \infty, L_p) \equiv -\Delta E_0(L_p)$, we must add to $d'(\infty, L_p)$ the stationary law $-\Delta E_{\text{coll}}(L_p)$ for purely collisional loss. For the latter we take the HTL result of [19] in the logarithmic accuracy $\log(k_{\max}/m_g) \gg 1$,
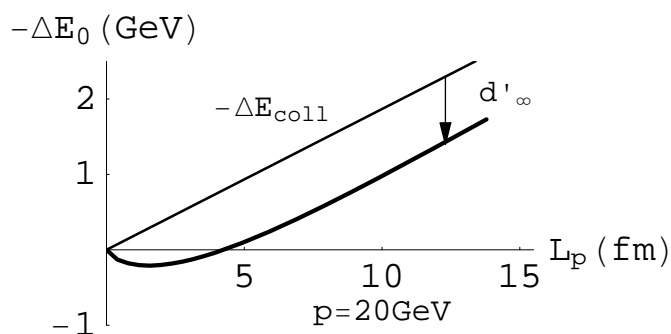
$$-\Delta E_{\text{coll}}(L_p) = \frac{C_F \alpha_s m_D^2 L_p}{2} \left[\frac{1}{v} - \frac{1 - v^2}{2v^2} \log\left(\frac{1 + v}{1 - v} \right) \right] \log\left(\frac{k_{\max}}{m_g} \right), \qquad (3.10)$$

where $k_{\max} \equiv \min\left\{ \frac{ET}{M}, \sqrt{ET} \right\}$ and $m_g = m_D/\sqrt{3}$.

The resulting energy loss

$$-\Delta E(L = \infty, L_p) = -\Delta E_{\text{coll}}(L_p) + d'(L = \infty, L_p)$$

---

[7]Strictly speaking, the effect of transition on $d'(L, L_p)$ starts at $L = L_s = 2vL_p/(1 + v) < L_p$, in agreement with causality, as shown in appendix B, see (B.1). For $p \geq 10\,\text{GeV}$, $v \simeq 1$ and the difference between L_s and L_p cannot be seen on figure 2.

Figure 3: Charm quark energy loss $-\Delta E_0(L_p) = -\Delta E(L = \infty, L_p)$ for $p = 20\,\text{GeV}$ (solid line) given by (3.11). $-\Delta E_0(L_p)$ is compared to the stationary law (3.10) for collisional energy loss $-\Delta E_{\text{coll}}(L_p)$ (straight line) [19].

$$= -\Delta E_{\text{coll}}(L_p) + d(L_p) - W'(L = \infty, L_p) \tag{3.11}$$

is determined by (2.9), (3.10) and (3.8), with the functions h_1 and h_2 given by (A.8) and (A.10).

Our result for $-\Delta E_0(L_p)$ is shown in figure 3. We observe an important suppression of the zeroth order energy loss as compared to the stationary collisional loss $-\Delta E_{\text{coll}}(L_p)$. This is mainly due to the fact that the asymptote d'_∞ of $d'(L = \infty, L_p)$, though reduced by a factor $1/3$ compared to d_∞ (see (3.9)), still scales as γ when $\gamma \gg 1$. As already mentioned in the end of section 2, this is due to a non-compensation between the TM effect and transition radiation. As is obvious from figure 3, we can define an *effective retardation time* t_{ret} before the linear regime for $-\Delta E_0(L_p)$ sets in,

$$t_{\text{ret}} = d'_\infty/(dE/dx)_{\text{coll}}, \tag{3.12}$$

leading to the scaling $t_{\text{ret}} \propto \gamma/m_D$, thus to a large effective delay. Numerically, $t_{\text{ret}} \simeq 5\,\text{fm}$ for $p = 20\,\text{GeV}$. Figure 3 is our main result, which shows that invoking only the heavy quark collisional energy loss $-\Delta E_{\text{coll}}(L_p)$ to explain the 'single electron puzzle' is not satisfactory. Indeed, adding the other relevant contributions (initial bremsstrahlung and transition radiation) to obtain the energy loss $-\Delta E_0(L_p)$ to zeroth order in an opacity expansion [6], we find that $-\Delta E_0(L_p) \ll -\Delta E_{\text{coll}}(L_p)$ for in-medium quark path length $L_p \lesssim 5 - 7$ fm.

4. Conclusion

In this paper we studied the energy loss $-\Delta E_0(L_p)$ of a fast heavy quark produced in a finite size QGP, to zeroth order in an opacity expansion. $-\Delta E_0(L_p)$ includes collisional energy loss, initial bremsstrahlung arising from the quark creation at $t = 0$, and transition radiation appearing when the quark passes the discontinuity between medium and vacuum. Only the radiative loss *induced by rescatterings* in the plasma $-\Delta E_{\text{rad}}(L_p)$ [5, 6] should be

added to $-\Delta E_0(L_p)$ to obtain the total heavy quark energy loss. Our main result is that $-\Delta E_0(L_p)$ is strongly reduced compared to the stationary linear law for collisional loss, as seen on figure 3. Due to the negative shift d'_∞ scaling as γ, the linear regime for $-\Delta E_0(L_p)$ sets in after a quite large effective retardation time $t_{\text{ret}} \propto \gamma$.

From figure 3 we conclude that for the generic values $L_p \sim 5$ fm and $p = 20\,\text{GeV}$, the loss $-\Delta E_0(L_p)$ should be negligible, and the total heavy quark energy loss should be theoretically correctly estimated by $-\Delta E_{\text{rad}}(L_p)$ only. Our results suggest that a missing contribution to the heavy quark energy loss, as proposed in ref. [11], might not be a correct explanation for the surprisingly strong nuclear attenuation of large $p_\perp$ electron spectra from heavy flavour decays.

We already mentioned in the Introduction that our results should be considered at the qualitative level, in particular because the slope $(-dE/dx)_{\text{coll}}$ of the stationary regime usually considered [16, 19] is modified when the running of α_s is taken into account [22]. As discussed in [15], the small L (small L_p) behaviour of figure 1 (figure 3) might also be affected by the running of α_s. However the position and scaling in γ of the asymptotes d_∞ and d'_∞ (see (3.9) and figure 3) should be unaffected by the running of the coupling [15], leaving unchanged the main result of the present study.

Acknowledgments

A. Derivation of the transition fields

In this appendix we obtain the functions $h_1(\omega, k_\perp)$ and $h_2(\omega, k_\perp)$ defining the transition fields (3.1). Written in the mixed space $(\omega, \vec{k}_\perp, z)$, the continuity conditions at the surface $z = L_p$ read:

$$\left[\vec{E}_1^{\text{in}}(\omega, \vec{k}_\perp, L_p) + \vec{E}_{T1}'(\omega, \vec{k}_\perp, L_p)\right]_\perp = \left[\vec{E}_2^{\text{in}}(\omega, \vec{k}_\perp, L_p) + \vec{E}_{T2}'(\omega, \vec{k}_\perp, L_p)\right]_\perp \quad \text{(A.1)}$$

$$\left[\vec{D}_1^{\text{in}}(\omega, \vec{k}_\perp, L_p) + \epsilon_1 \vec{E}_{T1}'(\omega, \vec{k}_\perp, L_p)\right]_z = \left[\vec{D}_2^{\text{in}}(\omega, \vec{k}_\perp, L_p) + \epsilon_2 \vec{E}_{T2}'(\omega, \vec{k}_\perp, L_p)\right]_z \quad \text{(A.2)}$$

where the color index is implicit and the inhomogeneous fields are given by $(i = 1, 2)$

$$\vec{E}_i^{\text{in}}(\omega, \vec{k}_\perp, z) = \int \frac{dk^z}{2\pi} e^{ik^z z} \left[\vec{E}_{Li}(\omega, \vec{k}_\perp, k^z) + \vec{E}_{Ti}(\omega, \vec{k}_\perp, k^z)\right] \quad \text{(A.3)}$$

$$\vec{D}_i^{\text{in}}(\omega, \vec{k}_\perp, z) = \epsilon_i(\omega) \vec{E}_i^{\text{in}}(\omega, \vec{k}_\perp, z). \quad \text{(A.4)}$$

Using the definition (3.1), we can easily solve for the functions h_1 and h_2. We find from the above continuity conditions:

$$h_1(\omega, k_\perp) = \frac{\epsilon_2 - \epsilon_1}{\epsilon_1 \sigma_2 + \epsilon_2 \sigma_1} [\epsilon_2 J + \sigma_2 K]$$

$$h_2(\omega, k_\perp) = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 \sigma_2 + \epsilon_2 \sigma_1} [-\epsilon_1 J + \sigma_1 K], \quad \text{(A.5)}$$

with

$$J = \frac{q^a \vec{k}_\perp \cdot \left[\vec{E}_2^{\,\text{in}\,a}(L_p) - \vec{E}_1^{\,\text{in}\,a}(L_p)\right]_\perp}{4\pi C_F \alpha_s k_\perp^2 (\epsilon_2 - \epsilon_1)} \quad \text{and} \quad K = \frac{q^a \left[\vec{D}_2^{\,\text{in}\,a}(L_p) - \vec{D}_1^{\,\text{in}\,a}(L_p)\right]_z}{4\pi C_F \alpha_s k_\perp^2 (\epsilon_2 - \epsilon_1)}\,. \quad \text{(A.6)}$$

Using (2.1) (with $\epsilon_{Li} = \epsilon_{Ti} = \epsilon_i$) and (2.2) we obtain:

$$J(\omega, k_\perp) = \frac{v}{\omega \epsilon_1 \epsilon_2} \int \frac{dk^z}{2\pi} \frac{e^{ik^z L_p} k^z \left[(\epsilon_1 + \epsilon_2)\omega^2 - k^2\right]}{(\omega - vk^z)(\epsilon_1 \omega^2 - k^2)(\epsilon_2 \omega^2 - k^2)}$$

$$K(\omega, k_\perp) = -v\omega \int \frac{dk^z}{2\pi} \frac{e^{ik^z L_p}}{(\omega - vk^z)(\epsilon_1 \omega^2 - k^2)(\epsilon_2 \omega^2 - k^2)}\,. \quad \text{(A.7)}$$

From (A.5) and (A.7) we readily check that $h_i(-\omega, k_\perp) = h_i(\omega, k_\perp)^*$. Hence the transition fields (3.1) satisfy $\vec{E}'_T(-\omega, -\vec{k}_\perp, z) = \vec{E}'_T(\omega, \vec{k}_\perp, z)^*$, ensuring the reality of the fields in coordinate space. We can further put (A.5) in the form

$$h_1(\omega, k_\perp) = i\frac{(\epsilon_2 - \epsilon_1)\omega}{\epsilon_1 \sigma_2 + \epsilon_2 \sigma_1} I_1$$

$$h_2(\omega, k_\perp) = i\frac{(\epsilon_1 - \epsilon_2)\omega}{\epsilon_1 \sigma_2 + \epsilon_2 \sigma_1} I_2\,, \quad \text{(A.8)}$$

where the functions I_1 and I_2 read

$$I_1 \equiv -\frac{iv}{\epsilon_1 \omega^2} \int \frac{dk^z}{2\pi} \frac{e^{ik^z L_p}}{(\omega - vk^z)(\epsilon_1 \omega^2 - k^2)} \left[\frac{k^z(k^z + \sigma_2) - \epsilon_1 \omega^2}{k^z + \sigma_2}\right]$$

$$I_2 \equiv -\frac{iv}{\epsilon_2 \omega^2} \int \frac{dk^z}{2\pi} \frac{e^{ik^z L_p}}{(\omega - vk^z)(\epsilon_2 \omega^2 - k^2)} \left[\frac{k^z(\sigma_1 - k^z) + \epsilon_2 \omega^2}{k^z - \sigma_1}\right] \quad \text{(A.9)}$$

We calculate I_1 and I_2 by closing the contour of the k^z-integral in the upper half of the complex plane. We obtain:

$$I_1 = \frac{1}{\epsilon_1 \omega^2} \left\{ \frac{e^{i\frac{\omega}{v}L_p}(\sigma_2 \frac{\omega}{v} - \epsilon_1 \omega^2 + \frac{\omega^2}{v^2})}{\left[\frac{\omega^2}{v^2} - \sigma_1^2\right]\left[\frac{\omega}{v} + \sigma_2\right]} + \frac{e^{i\sigma_1 L_p}(\sigma_1 \sigma_2 - k_\perp^2)}{2\sigma_1 \left[\sigma_1 - \frac{\omega}{v}\right](\sigma_1 + \sigma_2)} \right\} \quad \text{(A.10)}$$

$$I_2 = \frac{1}{\epsilon_2 \omega^2} \left\{ \frac{e^{i\frac{\omega}{v}L_p}(\sigma_1 \frac{\omega}{v} + \epsilon_2 \omega^2 - \frac{\omega^2}{v^2})}{\left[\frac{\omega^2}{v^2} - \sigma_2^2\right]\left[\frac{\omega}{v} - \sigma_1\right]} + \frac{e^{i\sigma_2 L_p}(\sigma_1 \sigma_2 + k_\perp^2)}{2\sigma_2 \left[\sigma_2 - \frac{\omega}{v}\right](\sigma_2 - \sigma_1)} + \frac{\epsilon_2 \, e^{i\sigma_1 L_p}}{(\epsilon_1 - \epsilon_2)\left[\sigma_1 - \frac{\omega}{v}\right]} \right\}$$

The transition fields (3.1) are determined by (A.8) and (A.10), where we recall that the retarded prescription $\omega \to \omega + i\eta$ is implicit, and that the square root satisfies $\text{Im}\sqrt{z} \geq 0$ for complex z.

B. Some properties of $-W'(L, L_p)$

Here we study some features of the contribution $-W'(L, L_p)$ to the heavy quark energy loss (see (3.8) and (3.11)) which arises from the transition fields (3.1). We first show that this contribution is consistent with causality. Then we derive, for $\gamma \gg 1$, the limit of $-W'(L, L_p)$ for $L \gg L_p \to \infty$. This limit corresponds to the amount to be added to the asymptotic value d_∞ of $d(L)$ to reach the asymptote d'_∞ of $d'(L)$ (see figure 2).

Causality. From causality the heavy quark cannot be sensitive to the transition fields right after its production at $t = 0$. The effect of transition fields can start only when a signal emitted by the quark at $t = 0$ (and $z = 0$) along the positive z-axis has had time to reach the medium-vacuum separation surface located at $z = L_p$, and to come back to the position of the quark. Since the quark is moving with velocity v, and the largest speed of light in a medium of permittivity (2.8) is unity (corresponding to $\omega \to \infty$), this time is given by $t_s = L_s/v = 2L_p - L_s$. Thus $-W'(L, L_p)$ must vanish when the quark has travelled less than the distance L_s,

$$L < L_s = \frac{2vL_p}{1+v} \Rightarrow -W'(L, L_p) = 0. \tag{B.1}$$

The latter causality requirement can be proved directly from (3.8). For $L < L_p$ only the first term in the bracket of (3.8) contributes. Inserting the expression of h_1 given by (A.8) and (A.10) we obtain:

$$-W'(L, L_p)\big|_{L<L_p} = \frac{C_F \alpha_s}{2\pi} \int_0^\infty dk_\perp^2 \, k_\perp^2 \int_{-\infty}^\infty \frac{d\omega}{\omega} \frac{e^{i(\frac{\omega}{v}+\sigma_1)L_p} - e^{i(\frac{\omega}{v}+\sigma_1)(L_p-L)}}{\frac{\omega}{v} + \sigma_1} \frac{1 - \epsilon_2/\epsilon_1}{\epsilon_1\sigma_2 + \epsilon_2\sigma_1} \times$$
$$\times \left\{ \frac{\sigma_2 \frac{\omega}{v} - \epsilon_1\omega^2 + \frac{\omega^2}{v^2}}{\left[\frac{\omega^2}{v^2} - \sigma_1^2\right]\left[\frac{\omega}{v} + \sigma_2\right]} + \frac{e^{i(\sigma_1 - \frac{\omega}{v})L_p}(\sigma_1\sigma_2 - k_\perp^2)}{2\sigma_1\left[\sigma_1 - \frac{\omega}{v}\right](\sigma_1 + \sigma_2)} \right\}. \tag{B.2}$$

Recalling (see section 3) that the square root satisfies $\operatorname{Im}\sqrt{z} \geq 0$ for complex z, we can perform the ω-integral by closing the integration contour in the upper half-plane provided the phase

$$2\sigma_1 L_p - \left(\frac{\omega}{v} + \sigma_1\right)L \tag{B.3}$$

has a positive imaginary part when $|\omega| \to \infty$ in this half-plane. Since for $\operatorname{Im}\omega > 0$ the integrand of (B.2) has no singularity (which can be easily checked) and $\sigma_1 \simeq \omega$ when $|\omega| \to \infty$, we obtain that $-W'(L, L_p)$ vanishes when $2L_p > (1 + 1/v)L$, which proves (B.1).

Limit of $-W'(L, L_p)$ when $L \gg L_p \to \infty$. We first consider the limit of $-W'(L, L_p)$ for $L \to \infty$ at fixed L_p. This represents the amount brought by transition radiation, to be added to $-\Delta E_{\text{coll}}(L_p) + d(L_p)$ (see (3.11)) in order to get the heavy quark energy loss $-\Delta E_0(L_p)$ represented in figure 3. For a fast quark, $\gamma \gg 1$, this limit is given by the second term of (3.8), where the term with a rapidly oscillating phase factor $\propto e^{-i(\frac{\omega}{v}-\sigma_2)L}$ is neglected:

$$-W'(L \to \infty, L_p) = \frac{C_F \alpha_s}{2\pi} \int_0^\infty dk_\perp^2 \, k_\perp^2 \int_{-\infty}^\infty d\omega \, \frac{e^{-i\frac{\omega}{v}L_p}(\epsilon_2 - \epsilon_1)\omega}{\left[\frac{\omega}{v} - \sigma_2\right](\epsilon_1\sigma_2 + \epsilon_2\sigma_1)} \, I_2, \tag{B.4}$$

where we used (A.8) and I_2 is given in (A.10).

We now evaluate the limit of (B.4) when $L_p \to \infty$. In this limit only the first term of I_2 in (A.10) contributes to (B.4), the two other terms oscillating rapidly. We get:

$$-W'(L, L_p)\big|_{L \gg L_p \to \infty} = \frac{C_F \alpha_s}{2\pi} \int_0^\infty dk_\perp^2 \, k_\perp^2 \int_{-\infty}^\infty \frac{d\omega}{\omega} \times$$

$$\times \frac{(1 - \epsilon_1/\epsilon_2)\left(\sigma_1 \frac{\omega}{v} + \epsilon_2\omega^2 - \frac{\omega^2}{v^2}\right)}{(\epsilon_1\sigma_2 + \epsilon_2\sigma_1)\left[\frac{\omega}{v} + \sigma_2\right]\left[\frac{\omega}{v} - \sigma_2\right]^2\left[\frac{\omega}{v} - \sigma_1\right]}\,. \quad \text{(B.5)}$$

The latter integral can be evaluated for $\gamma \gg 1$ by anticipating that it arises dominantly from the domain $\omega \sim \gamma m$ and $k_\perp \sim m$. Approximating the integrand in this region and using (2.8) (and $\epsilon_2 = 1$) we obtain

$$-W'(L, L_p)\big|_{L \gg L_p \to \infty} \underset{v \to 1}{\simeq} \frac{m^2 C_F \alpha_s}{\pi} \int_0^\infty dk_\perp^2 \, k_\perp^2 \int_{-\infty}^\infty \frac{d\omega}{(\frac{\omega^2}{\gamma^2} + k_\perp^2)^2(\frac{\omega^2}{\gamma^2} + k_\perp^2 + m^2)}\,. \quad \text{(B.6)}$$

The latter double integral can be exactly evaluated, yielding (use $m = m_D/\sqrt{2}$):

$$-W'(L, L_p)\big|_{L \gg L_p \to \infty} \equiv d'_\infty - d_\infty \underset{v \to 1}{\simeq} \frac{2}{3}\sqrt{2}C_F\alpha_s m_D\gamma\,. \quad \text{(B.7)}$$

The expression (B.6) is also trivially shown to be dominated by $\omega \sim \gamma m$, $k_\perp \sim m$, justifying our initial approximation. The result (B.7) is exactly $2/3$ of $|d_\infty|$, see (2.11). Hence:

$$d'_\infty \underset{v \to 1}{\simeq} \frac{1}{3}d_\infty \underset{v \to 1}{\simeq} -\frac{1}{3}\sqrt{2}C_F\alpha_s m_D\gamma\,. \quad \text{(B.8)}$$

Finally, we note that in (A.10) the second and third terms of I_2, which have been neglected to extract the large L_p limit of (B.4), are proportional to the phase factors $e^{-i(\frac{\omega}{v} - \sigma_{1,2})L_p}$. When $\omega \sim \gamma m \gg m$ and $k_\perp \sim m$ we can approximate, for instance,

$$\left(\frac{\omega}{v} - \sigma_1\right)L_p \simeq \frac{L_p}{2\omega}\left(\frac{\omega^2}{\gamma^2} + k_\perp^2 + m^2\right) \sim \mathcal{O}\left(L_p m/\gamma\right)\,, \quad \text{(B.9)}$$

Thus the large L_p limit used above should be understood as $L_p \gg \gamma/m_D$, where the neglected terms are indeed rapidly oscillating. In this limit the function $d'(L, L_p)$ is close to its asymptote d'_∞, i.e., the energy loss $-\Delta E_0(L_p)$ including the effect of transition radiation (see figure 3) is close to its asymptotic regime.

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